In the design of a radio circuit, one popular way to filter out unwanted signals is to use a super heterodyne receiver. One of the equations governing the operation of this device is the following:

$$\cos(a) \bullet \cos(b) = \frac{\cos(a+b) + \cos(a-b)}{2} \qquad (1)$$

This is useful because in order to make much of the radio signal, it first need to be frequency shifted down to a lower frequency. In order to do this, one just needs to multiply the incoming signal with a cosine wave whose frequency is equal to the amount you wish to frequency shift the signal. Is this a linear operation, i.e., is

$$y(t) = \cos(\omega t) \bullet x(t)$$

linear?

This a nonlinear circuit. Another problem comes into play in that it is not easy to multiply two signals together. This is solved by first adding the two signals together and passing the result through a non-linear device. The nonlinear devicecan be represented by the following:

$$y(t) = b + a_1 x(t) + a_2 x^2(t) + a_3 x^3(t)\ldots$$

For simplicity, assume that a_3 and above are 0.

Take the case where the input signal is $\cos(180 \bullet 10^6 \pi\ t)$. It is desired to bring this down to $(2 \bullet 10^6\ \pi\ t)$. By (1), this can be done by multiplying by $\cos(178 \bullet 10^6 \pi\ t)$. Using The nonlinear device approach, this will yield an output governed by the following equations:

$$x(t) = \cos\left(180 \bullet 10^6 \pi t\right) + \cos\left(178 \bullet 10^6 \pi t\right)$$

$$y(t) = b + a_1 x(t) + a_2 x^2(t)$$

$$y(t) = b + a_1\left(\cos\left(180 \bullet 10^6 \pi t\right) + \cos\left(178 \bullet 10^6 \pi t\right)\right) + a_2\left(\cos\left(180 \bullet 10^6 \pi t\right) + \cos\left(178 \bullet 10^6 \pi t\right)\right)^2$$

$$y(t) = b + a_1 \cos\left(180 \bullet 10^6 \pi t\right) + a_1 \cos\left(178 \bullet 10^6 \pi t\right) + a_2 \cos^2\left(180 \bullet 10^6 \pi t\right) + a_2 \cos^2\left(178 \bullet 10^6 \pi t\right) + \\ 2a_2 \cos\left(180 \bullet 10^6 \pi t\right)\cos\left(178 \bullet 10^6 \pi t\right)$$

$$y(t) = b + a_1 \cos\left(180 \bullet 10^6 \pi t\right) + a_1 \cos\left(178 \bullet 10^6 \pi t\right) + \frac{1}{2} a_2 \cos\left(360 \bullet 10^6 \pi t\right) + \frac{1}{2} a_2 \cos\left(356 \bullet 10^6 \pi t\right) + \\ a_2 \cos\left(2 \bullet 10^6 \pi t\right) + a_2 \cos\left(358 \bullet 10^6 \pi t\right)$$

The output has several frequency components. The desired $1 \bullet 10^6$ and the undesired: $89 \bullet 10^6$, $90 \bullet 10^6$, $118 \bullet 10^6$, $119 \bullet 10^6$, and $120 \bullet 10^6$. Design a filter that will reduce the $1 \bullet 10^6$ component by no more that 3db yet reduce all the others by at least 30db