In many signal-processing systems there is a transform called the Hilbert transform. It is often used in making a Single Sideband transmission, which allows one to transmit the same information using half the bandwidth. This transform has the input-output relationship that phase shifts every frequency by $\pi/2$. The output relationships is governed by the following equation for some constant C:

$$Y(\omega) = -j\,\text{sgn}(\omega) \bullet X(\omega)C$$

For this problem, do not concern yourself with negative frequency so limit yourself to:

$$Y(\omega) = -j \bullet X(\omega)C$$

Can you design a linear circuit that accomplishes this? Use phasors for your analysis.

If not, describe why not and create a design that approximates it for frequencies between 4kHz and 4.1kHz.