Time Value of Money Concepts: Interest Rates and NPV

Module 02.2: Interest and NPV

Revised: January 27, 2003

Purpose:

- Introduce Learners to the basic concepts of the "Time Value of Money."
- Make a point about the importance of this topic to all engineers and to your personal financial well being.
- Since, the text book for this course assumes that everyone has had a course in engineering economy … …

Learning Objective:

- Given a discrete future cashflow (a series of periodic cash payments and/or disbursements over time length N) compute the NPV (net present value) given the interest rate.
- Be able to draw a cashflow diagram of any given discrete cash stream.

Why This Is Important

- All Projects involve a cash stream of some sort.
  - It is usually a combination of both income and expenses.
  - If the net is positive the project "made money"; otherwise, it "lost money."
- One way to sort out project alternatives is through engineering economy.

The General Concepts

- Money, besides being a measure of value, is a commodity, just like gold, oil, wheat, pork bellies ... …
- It is can be bought, sold, borrowed, loaned, saved, consumed, and stolen.
- When money is borrowed the "rent" is called interest. If you loan money you earn interest; If you borrow money you pay interest.
- Because the amount of interest is a function of time, the value of an amount of money varies as a function of time – this is a new concept to most of you.

Concepts …

- There is simple interest and compound interest.
  - Simple interest is as old as history itself. It is simply a certain % of the money loaned. Time may, or may not, be a factor.
  - Compound interest is a relatively new invention (1700's?) and is essentially, interest on interest.

Other Essential Points You Need to Know…

- When interest rates are greater than zero, $$-amounts can only be summed at the same point in time.
- Usually, this means that all future $$ amounts are converted to a present value before they are summed.
- This is called "discounting" the cash flow.
- Almost every commercial project is evaluated and compared based upon some "discounted cashflow" – stocks, bonds, projects, real estate, … …

Other Points … …

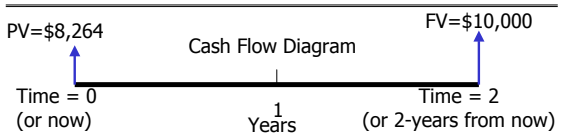
- When interest rates are **zero** $$-amounts can summed independent of time.
- Money is more valuable now than it is some time in the future -- **"Get the money up front!"**
- Unless specifically told otherwise, always assume compound interest.

The Basic Formula

- $PV = FV/(1+i\%)^{n}$
  - PV or P is present value
  - FV or F is some amount in the future
  - i%= the interest rate per period, years, months, weeks,
  - n = the number of periods

Example # 1 – Single Amount

- Question: What is the PV (the value now) of $10,000 that you expect to receive 2 years from now, if current interest rates are 10% compounded annually?
- Answer: $PV=\$10,000/1.1^{2} = \$8,264$



RAT #3.1.1 – Take Up

- Work a P = F(1+%)^-n problem
- Work a F = P(1+%)^n problem
- As Individuals

Example #2 - Multiple Amounts

Discount given cash stream @ 10%

EOY	Amount	1/(1+%)^n	PV
0	-10,000	1.0000	-10,000
1	2,000	0.9091	1,818
2	3,000	0.8264	2,479
3	4,000	0.7513	3,005
4	5,000	0.6830	3,415
10%	4,000	<- Sums ->	718

The Discount Factor is: $(1+i)^{n} = 1.1^{n}$

RAT #3.1.2 Data

Compute the Present Value, if i=0% (individuals) and i=20%. (team)

EOY	Amount	Disc. Factor	PV
0	-$10,000		
1	$2,000		
2	$3,000		
3	$7,000		
Total			

Memorize these Basic Assumptions to Avoid Exam Mistakes.

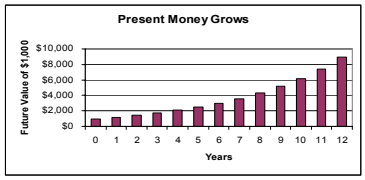
- The time the money is loaned or borrowed is broken into even time intervals (or, periods) – years, quarters, months, days.
- All cash-flow events occur at the ends of the time intervals and the interest rate per period is constant.
- Interest rates are generally expressed as nominal annual (per year =12%) but must be adjusted to fit the compounding period (per month =1%, per quarter =3%). A very common exam mistake.

$1,000 now is equivalent to $8,916 12-years in the future at 20% interest.

Period	20%
0	$1,000
1	$1,200
2	$1,440
3	$1,728
4	$2,074
5	$2,488
6	$2,986
7	$3,583
8	$4,300
9	$5,160
10	$6,192
11	$7,430
12	$8,916

Brute Force

$8,916 =F =P(1+%)^{n} =$1,000(1.2)^{12}



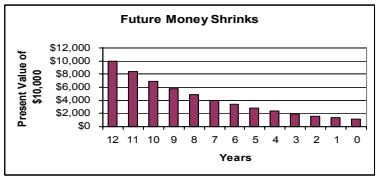
Maxwell's 1-st Law:
Get the Money Up-Front

$10,000 12-years in the future at 20% is equivalent to $1,122 now.

Period	20%
12	$10,000
11	$8,333
10	$6,944
9	$5,787
8	$4,823
7	$4,019
6	$3,349
5	$2,791
4	$2,326
3	$1,938
2	$1,615
1	$1,346
0	$1,122

Brute Force

$1,122 =P =F(1+%)$^{-n}$ =$10,000(1.2)$^{-12}$



Maxwell's Other Law:
Take the Money and Run!

Summary

- One way to evaluate projects, stocks, bonds, etc. is by discounted cash flow.
- Amounts of money scattered at various points in time can only be summed at the same point in time – usually now.
- The relationship: $PV=FV/(1+\%)^{n}$ is used to "move" money from one point in time to another